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Simulation of Pull-out Torque of Hybrid Stepping Motor

J. U. Agber and C. O. Onah

Abstract- Several models for predicting hybrid stepping motor characteristics have been developed. However most of these are based on linear theory, which do not consider the motor as a toothed structure working under highly saturated conditions. This paper considers the hybrid permanent magnet stepping motor as a non-linear device. It approximates the measured flux-linkage data with analytical function that has continuous derivatives. The electrical and mechanical equations for the motor are derived and used in a computer program to predict the pull-out torque versus speed characteristics. The comparisons of experimental and simulated results show good agreement.

Key words: Pull-out torque, Steady-state characteristics, Hybrid stepping motor, Permanent magnet motor, Doubly salient device.

1 INTRODUCTION

The inherent and reliable positioning of loads has led to increased demand on the use of stepping motors in terminal devices and hence interest in their dynamic characteristics. Snowden and Madsen [1] carried out the first detailed attempt, in which the synchronous inductor motor was analyzed using the equivalent circuit of a synchronous motor and a second-order linear model was used to represent it. Other models have also been developed by various authors [2, 3]. However, these models are based on the assumption that the stepping motors have a smooth air gap, and ignore the fact that the motors are doubly salient devices working under considerable saturation. Whilst approximate analysis can be useful for indicating the general trends in the dynamic behaviour, there is a need for models to be capable of representing accurately the electromagnetic conditions existing within the machine at all practical levels of excitation [4 – 8]. This need extends to representing adequately the magnetic non-linearity.

Before any stepping motor dynamics can be simulated, some basic information regarding the system and the simulation procedure is needed. These include the initial conditions and a system of differential equations. As far as the terminal equations are concerned, the motors can often be regarded as multi-pole synchronous machines [1, 2, 3, 8].

This paper is an extension of the analytical method for prediction of static characteristics of the permanent magnet (PM) hybrid stepping motor [9] to the prediction of pull-out torque characteristics and shows that the method is capable of producing high accuracy of prediction.

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2 FORMULATION OF THE METHOD

2.1 The voltage equation

The flux-linkage of a winding *k* in an angular rotational electromechanical system may be written as a function of the winding current, *i* and the angular position, θ, of the rotating member.

$$\phi_k = \phi(i,\theta) \tag{1}$$

Taking the partial derivatives

$$d\phi_k = \frac{\partial \phi(i,\theta)}{\partial i} di + \frac{\partial \phi(i,\theta)}{\partial \theta} d\theta \tag{2}$$

The induced voltage, E_k, in such a winding may be written by Faraday's Law.

$$E_k = \frac{d\phi_k}{dt} = \frac{\partial \phi(i,\theta)}{\partial i}\frac{di}{dt} + \frac{\partial \phi(i,\theta)}{\partial \theta}\frac{d\theta}{dt} \tag{3}$$

The terminal voltage of such a system, represented in Figure 1, can then be written as:

$$V_k = iR + E_k = iR + \frac{d\varphi_k}{dt} \tag{4}$$

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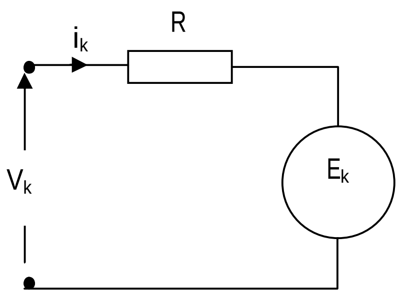

Fig.1. The equivalent circuit diagram of one phase of a hybrid stepping motor

By substituting equation (3) into (4) for $d\varphi_k / dt$, a general voltage equation is obtained.

$$V_k = iR + \frac{\partial\phi(i,\theta)}{\partial i}\frac{di}{dt} + \omega\frac{\partial\phi(i,\theta)}{\partial\theta} \quad (5)$$

where $\omega = d\theta / dt$ – the angular velocity of the shaft.

Equations (4) and (5) are the general forms of voltage equation for one phase of a stepping motor. The third term on the right of equation (5) is the motional emf (*e*). In the PM hybrid motor, this term can be split into two components as shown in equation (6).

$$e = \omega\left[\frac{d\phi_{pm}(\theta)}{d\theta} + \frac{\partial\phi(i,\theta)}{\partial\theta}\right] \quad (6)$$

The first term on the right is the motional emf produced by the PM flux, while the second is due to stator excitation.

It has been shown [10] that the electromagnetic torque $T(i,\theta)$ of a system can be calculated from its magnetic coenergy, $W'(i,\theta)$, if the flux-linkage $\varphi_T(i,\theta)$ is known (see equations (7) and (8).

$$T(i,\theta) = \frac{\partial W(i,\theta)}{\partial\theta} \quad (7)$$

and

$$W(i,\theta) = \int_0^i \phi_T(i',\theta)di' \quad (8)$$

where *i'* is a dummy variable for current.

It has also been shown [9] that the total flux-linkage, $\varphi_T(i,\theta)$, of the hybrid stepping motor can be represented by analytical function.

$$f_T(i,\theta) = A_{00} + \sum_{n=1}^{3} A_{0n} Cos(nN_r\theta) + \sum_{k=0}^{5}\sum_{j=1}^{9} A_{jk} i^j Cos(kN_r\theta) \quad (9)$$

Where j = 1, 3, …, 9 (an odd integer), k = 1, 2, …, 5, n = 1, 3 (an odd integer) and N_r is the number of rotor teeth.

The first and second terms on the right of equation (9) represent the PM flux function, while the third represents the current dependent, i.e., the flux function due to stator excitation.

2.2 The dynamic equation of the system

The general voltage equation for the system has been derived in equation (5). In the dynamic case interest is centred on the rate of change of the phase current with time. By rearranging equation (5), an equation defining this rate of change can be obtained, thus

$$\frac{di}{dt} = \frac{V - iR - \omega\left[\partial\phi(i,\theta)/\partial\theta\right]}{\partial\phi(i,\theta)/\partial i} \quad (10)$$

The electrical equation for phase A can be written as follows:

$$\frac{di_a}{dt} = \frac{V - i_a R - \omega\left[\partial\phi(i_a,\theta_a)/\partial\theta_a\right]}{\partial\phi(i_a,\theta_a)/\partial i_a} \quad (11)$$

Similar expression exists for phase B by changing the indices *a* to *b*.

The mechanical equations are:

$$\frac{d\theta}{dt} = \omega \quad (12)$$

$$\frac{d\omega}{dt} = \frac{1}{J}\left[T(i_k,\theta_k) - K_v\omega - T(sign\omega)\right] \quad (13)$$

where:

$\theta_a = \omega t$,

$\theta_b = \omega t - \pi / 2$ ……… angular displacement of phases and

K_v – coefficient of viscous friction.

$$T(i_k,\theta_k) = T(i_a,\theta_a) + T(i_b,\theta_b) \quad (14)$$

Equations (10) to (13) are the dynamic equations of the hybrid stepping motor. For the steady state operation, it is assumed that the velocity of the rotor is constant and the speed is defined by the frequency of the applied terminal voltage. The dynamic equations are reduced to only the electrical equations, i.e. equations (10) and (11). These equations contain the derivatives $\partial\varphi(i,\theta)/\partial i$ and $\partial\varphi(i,\theta)/\partial\theta$, which can readily be obtained by differentiating the total flux-linkage expression in equation (9). The pull-out torque is found from equations (7) and (14).

The solution of the steady-state equations (10 and11) and pull-out torque, equation (14), is implemented in Matlab environment and the results are presented in Figures 2 and 3 for 115 and 230V supplies.

3 Discussion

Stepping motor applications have come to stay and hence the need for accurate prediction of their characteristics. The present work is an attempt to contribute to the numerous methods developed by other scholars. This method uses polynomial function to represent measured flux-linkage data. The flux function is integrated with respect to current to produce the systems coenergy, which when differentiated with respect to rotor angular position yields electromagnetic torque. The systems electrical and mechanical equations are used to predict the necessary characteristics and the graphical representation of the measured and predicted results have been presented.

Figures 3 and 4 show the pull-out torque vs. rotor angular position characteristics obtained for both 115V/10.0A and 230V/7.5A supply limits respectively. Good agreement between the measured and simulated results is obtained for both supply voltages. A typical discrepancy of 3.6% exists throughout the frequencies covered, except for regions of instability where it is as high as 30%. These regions of instability need special attention, which is not covered in the present work.

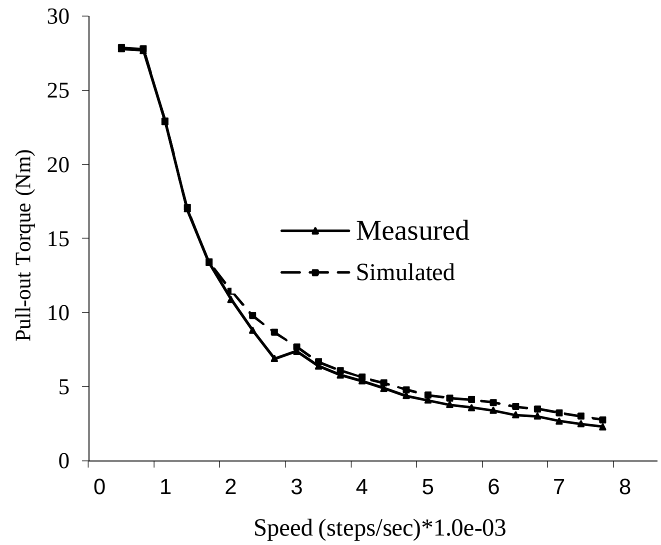

Fig.3. Measured and predicted pull-out torque vs speed characteristics using 115V/10.0A supply limit

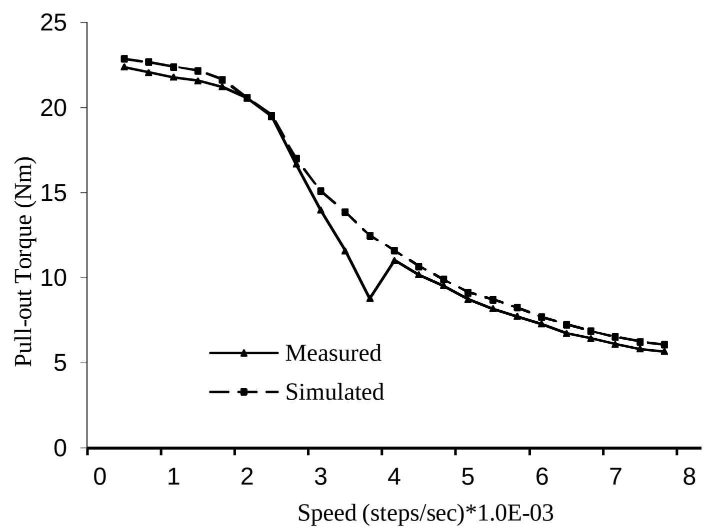

Fig. 4. Measured and predicted pull-out torque vs speed characteristics using 230V/7.5A supply.

4 Conclusion

A non-linear analytical method for predicting the pull-out torque characteristics of the hybrid stepping motor using measured flux-linkage data has been presented. The method uses analytical functions in current *i*, and trigonometrical functions in rotor angular position θ, to represent the flux-linkage data. The implementation of the computer program produces results that are in good agreement with the experimental data.

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